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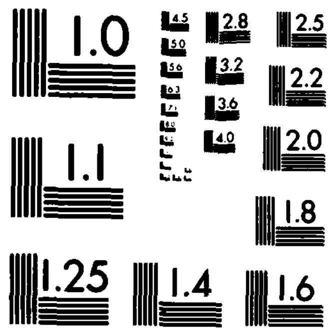

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February 1983

ON THE ODD-SYMMETRY OF MINIMUM PHASE-ONLY PERTURBATIONS

Robert A. Shore

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441

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It is proved that the minimum phase-only weight perturbations required to impose a single null in a real, linear array, antenna pattern are odd-symmetric with respect to a phase reference at the center of the array. Hence the perturbed pattern is likewise real. Additional evidence is given for supporting the hypothesis that the minimum phase-only weight perturbations are odd-symmetric for any number of imposed nulls.

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Preface

The author wishes to thank Dr. Hans Steyskal for many helpful discussions.

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Illustrations

On the Odd-Symmetry of Minimum Phase-Only Perturbations

I. INTRODUCTION

The increasing importance of phased arrays has been accompanied by considerable interest in phase-only control of array element weights for adaptive nulling[1-5] and for null synthesis.[6-13] Unlike for combined phase and amplitude control, the problem of imposing nulls in a linear array pattern with phase-only weight control is nonlinear and cannot be solved exactly analytically. Approximations and/or numerical techniques must be employed to calculate the required phase perturbations.

A problem of some practical interest in null synthesis is that of imposing nulls in a given array antenna pattern at a prescribed set of locations in such a way as to minimize the effect on the given pattern at other locations. If the given pattern is real and the weight perturbations are of the element phases only, it is intuitive* that the perturbed pattern should likewise be real and hence that the phase perturbations be odd-symmetric with respect to a phase reference taken at the center of

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*In their basic paper on adaptive nulling with phase-shifters, Baird and Rassweiler[1] make an implicit assumption based on intuition that the optimal phase-only weights for suppressing noise from a set of specified directions are odd-symmetric. The coefficients γ_k in their Eq. (8) are implicitly assumed to be real.

(Due to the large number of References cited above, they will not be listed here. See References, page 15.)

the array. If numerical techniques such as nonlinear programming are used to compute the desired phase perturbations, the odd-symmetry property can be used to reduce the number of independent unknown phases by a factor of a half. Although the odd-symmetry property of the minimum phase-only perturbations may be intuitive, nevertheless a proof of this property would be desirable.

The principal object of this report is to prove the odd-symmetry property of the minimum phase-only weight perturbations required to impose a single null in a real array antenna pattern. The proof unfortunately does not appear to lend itself to generalization to the case of more than one null. However, we present additional evidence to support the conjecture that the minimum phase-only perturbations are in general odd-symmetric, and it is hoped that this paper will serve as a stepping stone to the finding of a general proof.

2. ANALYSIS

Consider a linear array of equispaced isotropic elements (see Figure 1). The spacing between the elements is d and the phase reference center is taken to be at the center of the array. Let a_n, $n = 1, 2, \ldots, N$, be the amplitude of the nth array element, and assume symmetry of the amplitudes with respect to the reference center; that is,

$$a_n = a_{N-n+1}, \quad n = 1, 2, \ldots, N . \tag{1}$$

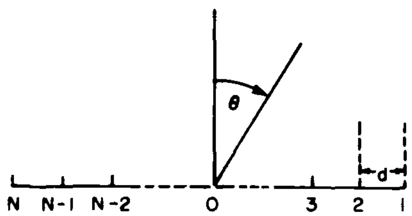

Figure 1. Geometry of Array

Then the array field pattern, $p_o(u)$, is

$$p_o(u) = \sum_{n=1}^{N} a_n e^{jd_n u}$$

where

$$d_n = \frac{N-1}{2} - (n-1), \quad n = 1, 2, \ldots, N$$

and

$$u = kd \sin\theta$$

with

$$k = \frac{2\pi}{\lambda}$$

and θ the angle measured from broadside to the array. The $\{d_n\}$ are odd-symmetric with respect to the phase reference center; that is,

$$d_n = -d_{N-n+1},\ n = 1, 2, \ldots, N\ . \tag{2}$$

The pattern $p_o(u)$ is real because of the symmetry of the $\{a_n\}$ and the $\{d_n\}$.

Now suppose it is desired to impose a null in the pattern at the location $u = u_1$ with phase-only perturbations $e^{j\phi_n}$ of the array weights. That is, we would like to have

$$\sum_{n=1}^{N} a_n e^{j\phi_n} e^{j d_n u_1} = 0\ . \tag{3}$$

At the same time we wish to minimize the weight perturbations in a least squares sense. Since the perturbation of the nth weight is $a_n(e^{j\phi_n} - 1)$, we wish to minimize the quantity

$$\sum_{n=1}^{N} a_n^2 \left|e^{j\phi_n} - 1\right|^2 = 4 \sum_{n=1}^{N} a_n^2 \sin^2\left(\frac{\phi_n}{2}\right). \tag{4}$$

We claim that the desired phase perturbations are then odd-symmetric with respect to the phase reference center; that is,

$$\phi_n = -\phi_{N-n+1},\ n = 1, 2, \ldots, N \tag{5}$$

so that the resulting field pattern remains real. The procedure we will use to prove Eq. (5) is first to assume that a set of phase perturbations has been found to satisfy Eq. (3); then to express these phase perturbations as a sum of two sets of perturbations, one with even-symmetry and the other with odd-symmetry; and finally, to show that a set of odd-symmetric phase perturbations can be constructed that satisfies Eq. (3) and yields a lower value of the quantity

$$\sum_{n=1}^{N} a_n^2 \sin^2\left(\frac{\phi_n}{2}\right)$$

than the starting set of phase perturbations with an even-symmetric component.

Accordingly, we begin with a set of phase perturbations, ϕ_n, $n = 1, 2, \ldots, N$, that satisfy Eq. (3), and let

$$\phi_n = \phi_{ne} + \phi_{no}, \quad n = 1, 2, \ldots, N \tag{6}$$

where*

$$\left.\begin{aligned} \phi_{ne} &= \frac{1}{2}(\phi_n + \phi_{N-n+1}) \\ \phi_{no} &= \frac{1}{2}(\phi_n - \phi_{N-n+1}) \,. \end{aligned}\right\} \quad n = 1, 2, \ldots, N$$

Then

$$\left.\begin{aligned} \phi_{N-n+1,e} &= \phi_{ne} \\ \phi_{N-n+1,o} &= -\phi_{no} \end{aligned}\right\} \quad n = 1, 2, \ldots, N \tag{7}$$

so that the $\{\phi_{ne}\}$ are the even-symmetric part and $\{\phi_{no}\}$ the odd-symmetric part of the phase perturbations $\{\phi_n\}$. Substituting Eq. (6) in Eq. (3),

$$\sum_{n=1}^{N} a_n e^{j(\phi_{ne} + \phi_{no})} e^{j d_n u_1} = 0$$

from which it is straightforward to obtain, utilizing the symmetries of Eqs. (1), (2), and (7), the equation

$$\sum_{n=1}^{N} a_n \cos\phi_{ne} \cos(d_n u_1 + \phi_{no}) + j \sum_{n=1}^{N} a_n \sin\phi_{ne} \cos(d_n u_1 + \phi_{no}) = 0 \,. \tag{8}$$

We now note that a purely odd-symmetric solution $\{\phi'_{no}\}$ of Eq. (3) can be constructed by defining ϕ'_{no} to be the solution of the equation

---

*If the number of elements in the array is odd, the phase of the center element is, of course, zero since this element is the phase reference for the array.

$$\cos \phi_{ne} \cos(d_n u_1 + \phi_{no}) = \cos(d_n u_1 + \phi'_{no}), \quad n = 1, 2, \ldots, N \tag{9}$$

with the smallest absolute value. (It is clear that a solution of Eq. (9) always exists since the left hand side of Eq. (9) has magnitude less than or equal to one.) For the set of phase perturbations so constructed, the terms of the real part of Eq. (8) remain unchanged and so their sum continues to equal zero, while the imaginary part of Eq. (8) is zero because all the $\{\phi'_{ne}\}$ are identically equal to zero. To show that the set of odd-symmetric phase perturbations obtained by solving Eq. (9) gives a smaller value of the quantity

$$\sum_{n=1}^{N} a_n^2 \sin^2\left(\frac{\phi_n}{2}\right)$$

than the starting set of phase perturbations with both an even- and odd-symmetric component, we first note that in view of Eq. (7) it is sufficient to show, taking the elements of the array by symmetrically located pairs, that

$$\sin^2\left(\frac{\phi_{ne} + \phi_{no}}{2}\right) + \sin^2\left(\frac{\phi_{ne} - \phi_{no}}{2}\right) \geq 2 \sin\left(\frac{\phi'_{no}}{2}\right). \tag{10}$$

Using the trigonometric identity

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2}(1 - \cos\theta)$$

and the formulas for the cosine of a sum and difference of two angles, it is simple to show that the inequality, (10), is equivalent to the inequality

$$\cos \phi_{ne} \cos \phi_{no} \leq \cos \phi'_{no} . \tag{11}$$

In Appendix A we show that the solution ϕ'_{no} of Eq. (9) with the smallest absolute value satisfies the inequality, (11), (strict inequality holds unless $d_n u_1$ is an integral multiple of π) and thus complete the proof that the minimum phase-only perturbations that result in an imposed null at a prescribed location in a real pattern have odd-symmetry with respect to the reference center of the array.

3. DISCUSSION

The proof of the odd-symmetry property of the minimum phase-only perturbations we have given above is based on the assumption that there is only one null to

be imposed in the antenna pattern. If there is more than one imposed null, the procedure of the proof does not appear to be capable of generalization. Suppose, for example, that nulls are to be imposed at two locations, u_i, i = 1, 2. Then, corresponding to Eq. (8), we now have the pair of equations

$$\sum_{n=1}^{N} a_n \cos \phi_{ne} \cos(d_n u_i + \phi_{no}) + j \sum_{n=1}^{N} a_n \sin \phi_{ne} \cos(d_n u_i + \phi_{no}) = 0$$
$$i = 1, 2$$

and there is no way in general that ϕ'_{no} can be found to simultaneously satisfy the pair of equations

$$\cos \phi_{ne} \cos (d_n u_i + \phi_{no}) = \cos(d_n u_i + \phi'_{no}), \; i = 1, 2$$

as required by the proof [see Eq. (9)]. It is true that we can find a different set of odd-symmetric phase perturbations $\phi'_{no,i}$, i = 1, 2 to satisfy each of the equations individually, but the nonlinearity of the phase-only nulling equation, Eq. (3), then does not allow any simple way of combining the two sets of odd-symmetric phase-only perturbations, one for each null, to give one set of phase perturbations that will result in nulls at both of the locations simultaneously.

Although the proof we have given cannot be directly generalized to the case of more than one imposed null, additional evidence can be cited to support the conjecture that the minimum phase-only perturbations required to impose nulls in a real pattern at a prescribed set of locations have odd-symmetry. For one thing, Shore and Steyskal[9] have shown that if the assumption of small phase perturbations is made and used to linearize the phase-only nulling equation

$$\sum_{n=1}^{N} a_n e^{j\phi_n} e^{j d_n u_i} = 0, \; i = 1, 2, \ldots, M,$$

the resulting set of equations can be solved exactly analytically subject to the constraint that the weight perturbations be minimized in a generalized least squares sense. The phases thus obtained are odd-symmetric with respect to the reference center located at the center of the array.

Additional evidence to support the odd-symmetry hypothesis comes from the use of nonlinear programming techniques[14] to calculate the minimum phase-only

14. Shore, R. (to be published, 1983), Phase-Only Nulling as a Nonlinear Programming Problem, RADC-TR-83-37.

perturbations required to impose nulls in a pattern at a prescribed set of locations. It is found that even if no symmetry requirement is placed on the solution so that there are N unknown phases for an array of N elements, N even, (N - 1 if N is odd) the solution obtained has odd-symmetry and is identical to that obtained if the assumption of odd-symmetry is made at the outset to reduce the number of unknown phases by a factor of one half. A sample output is given in Appendix B.

In further support, in Appendix C it is shown, using the Lagrange multiplier method, that the minimum phase perturbations satisfy the equation

$$\sum_{n=1}^{N} a_n \sin \phi_n = 0$$

a property consistent with the odd-symmetry hypothesis in view of the even-symmetry of the $[a_n]$ assumed throughout [see Eq. (1)].

Thus it appears likely, although as yet unproven in general, that the odd-symmetry hypothesis is true, and it is hoped that this report will stimulate further effort to find a general proof.

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Appendix A

Proof of a Trigonometric Inequality

In Appendix A we prove that the solution u to the equation

$$\cos x \cos (z + y) = \cos (z + u) \tag{A1}$$

with the smallest absolute value satisfies the inequality

$$\cos x \cos y \leq \cos u \tag{A2}$$

with strict inequality holding for all values of z not an integral multiple of π. Let x and y be arbitrary angles and consider the behavior of u as a function of z for z in the interval $[-\pi, \pi]$. Defining $\cos^{-1}(\theta)$ to lie in the interval $[0, \pi]$, we see that there are two solutions of Eq. (A1)

$$u_1 (z) = \cos^{-1} [\cos x \cos (z + y)] - z \tag{A3a}$$

and

$$u_2 (z) = -\cos^{-1} [\cos x \cos (z + y)] - z \ . \tag{A3b}$$

Since

$$u_2(-z) = u_1(\pi - z)$$
$$u_1(-z) = u_2(\pi - z) \tag{A4}$$

it suffices to consider the behavior of $u_1(z)$ and $u_2(z)$ for z in the interval $[0, \pi]$. As z increases from 0 to π, $\cos x \cos(z + y)$ varies from $\cos x \cos y$ to $\cos x \cos(\pi + y) = -\cos x \cos y$ and correspondingly, $\cos^{-1}[\cos x \cos(z + y)]$ varies from $\cos^{-1}(\cos x \cos y)$ to $\cos^{-1}(-\cos x \cos y) = \pi - \cos^{-1}(\cos x \cos y)$. Hence $u_1(z)$ varies from $\cos^{-1}(\cos x \cos y)$ to $-\cos^{-1}(\cos x \cos y)$ while $u_2(z)$ varies from $-\cos^{-1}(\cos x \cos y)$ to $\cos^{-1}(\cos x \cos y) - 2\pi$. Furthermore, since

$$\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1 - x^2}} \quad ,$$

differentiating Eqs. (A3a, A3b),

$$u'_{\substack{1\\2}}(z) = \pm \frac{\cos x \sin(z + y)}{\sqrt{1 - \cos^2 x \cos^2(z + y)}} - 1 \, .$$

But

$$\frac{\cos^2 x \sin^2(z + y)}{1 - \cos^2 x \cos^2(z + y)} = \frac{\cos^2 x - \cos^2 x \cos^2(z + y)}{1 - \cos^2 x \cos^2(z + y)} \leq 1$$

with equality if and only if $\cos^2 x = 1$. Hence if $x \neq N\pi$, $N = 0, \pm 1$, both $u_1'(z)$ and $u_2'(z)$ are negative over the entire range of z and so $u_1(z)$ and $u_2(z)$ are monotonically decreasing functions of z. It follows that as z increases from 0 to π, $u_1(z)$ decreases monotonically from $\cos^{-1}(\cos x \cos y)$ to $-\cos^{-1}(\cos x \cos y)$ while $u_2(z)$ decreases monotonically from $-\cos^{-1}(\cos x \cos y)$ to $\cos^{-1}(\cos x \cos y) - 2\pi$, (see Figure A1). Hence, in the interval $0 \leq z \leq \pi$, $u = u_1(z)$ is the solution of Eq. (A1) with the smallest absolute value and takes on its largest absolute values at the endpoints of this interval. Since at the endpoints $\cos u = \cos x \cos y$, it follows that for all values of z in between, $\cos x \cos y < \cos u$ and so we have demonstrated inequality (A2). If $x = 0$

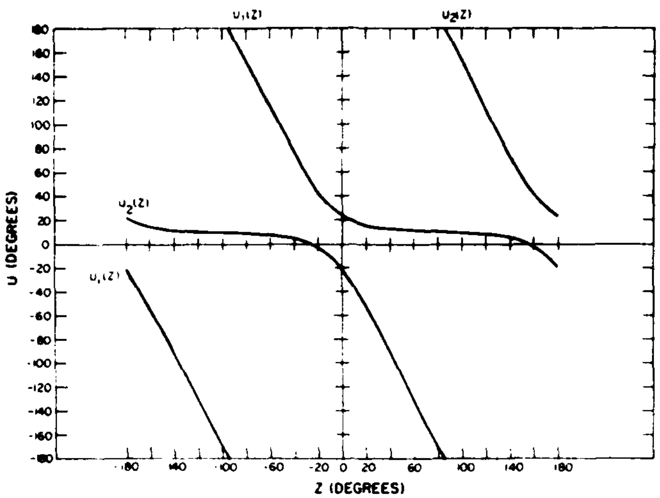

Figure A1. Plot of $u_{\substack{1\\2}}(z) = \pm \cos^{-1}[\cos x \cos(z + y)] - z$, $x = 20°$, $y = 10°$

$$u_1'(z) = \begin{cases} 0, & z + y \text{ in 1st and 2nd quadrants} \\ -2, & z + y \text{ in 3rd and 4th quadrants} \end{cases}$$

$$u_2'(z) = \begin{cases} -2, & z + y \text{ in 1st and 2nd quadrants} \\ 0, & z + y \text{ in 3rd and 4th quadrants} \end{cases}$$

while if $x = \pm\ \pi$,

$$u_1'(z) = \begin{cases} 0, & z + y \text{ in 3rd and 4th quadrants} \\ -2, & z + y \text{ in 1st and 2nd quadrants} \end{cases}$$

$$u_2'(z) = \begin{cases} -2, & z + y \text{ in 3rd and 4th quadrants} \\ 0, & z + y \text{ in 1st and 2nd quadrants} \end{cases}$$

and the inequality (A2) remains true as before, the only difference being that equality holds over an interval of z instead of at just the endpoints of the interval $0 \leq z \leq \pi$. In the interval $-\pi \leq z \leq 0$, we choose $u = u_2(z)$ and use Eq. (A4) and the above results for $0 \leq z \leq \pi$ to demonstrate inequality (A2), (see Figure A1). Thus the proof is complete.

Appendix B

Minimum Phase Perturbations Calculated by Nonlinear Programming

As an example of the odd-symmetry of the phases obtained as a nonlinear programming solution of the minimum phase-only weight perturbation problem, we show results for a 21-element uniform amplitude array with half-wavelength spacing The locations of the imposed nulls are 12, 15, and 18 degrees. The nonlinear programming computer code LPNLP[15] was used. When no symmetry requirement was imposed on the solution, the phases obtained are (in radians)

X VALUES, X(1),..., X(N)

0.568771D+00	0.369075D+00	-0.275962D+00	-0.329874D+00	-0.146163D+00
0.512906D-01	0.152084D+00	0.695103D-01	-0.890314D-01	-0.103123D+00
0.103123D+00	0.890314D-01	-0.695103D-01	-0.152084D+00	-0.512906D-01
0.146163D+00	0.329874D+00	0.275962D+00	-0.369075D+00	-0.568771D+00

The first two rows are ϕ_1 through ϕ_{10} while the second two rows are ϕ_{12} through ϕ_{21}. ($\phi_{11} = 0$ since the 11th element is the phase reference.) The odd-symmetry of the solution is apparent. If odd-symmetry is imposed as a requirement on the solution from the outset and the number of unknown phases halved, the output is

15. Pierre, D.A., and Lowe, M.J. (1975) Mathematical Programming Via Augmented Lagrangians: An Introduction With Computer Programs Addison-Wesley Publishing Co., Reading, Massachusetts.

```
X VALUES,  X(1),...,X(N)
  0.568771D+00  0.369075D+00  -0.275962D+00  -0.329874D+00  -0.146163D+00
  0.512906D-01  0.152084D+00   0.695103D-01  -0.890314D-01  -0.103123D+00
```

which agrees perfectly with the former solution. The null depths corresponding to these solutions are < -220 dB.

Appendix C

Proof That the Minimum Phase Perturbations Satisfy the Equation $\sum_{n=1}^{N} a_n \sin \phi_n = 0$

In Appendix C we use the Lagrange multiplier technique to show that the minimum phase perturbations required to place nulls at the set of prescribed pattern locations u_k, $k = 1, 2, \ldots, M$, have the property that

$$\sum_{n=1}^{N} a_n \sin \phi_n = 0 \,. \tag{C1}$$

From the theory of constrained optimization,[16] a necessary condition for a local minimum of a function $f(\underline{x})$ subject to a set of equality constraints $c_k(\underline{x}) = 0$, $k = 1, 2, \ldots, M$, is that the gradient of f be a linear combination of the gradients of the constraint functions; that is

$$\nabla f(\underline{x}^*) = \sum_{k=1}^{M} \lambda_k \nabla c_k(\underline{x}^*) \tag{C2}$$

where $\underline{x}^*$ denotes the value of the vector of variables at the local minimum. The multipliers in this linear combination are referred to as Lagrange multipliers and the superscript * indicates that they are associated with the point $\underline{x}^*$.

16. Fletcher, R. (1981) Practical Methods of Optimization. Volume 2. Constrained Optimization. John Wiley and Sons, New York.

Here

$$f(\underset{\sim}{\phi}) = \sum_{n=1}^{N} a_n \sin^2\left(\frac{\phi_n}{2}\right)$$

and the constraints are [see Eq. (3)]

$$\left.\begin{aligned} &\sum_{n=1}^{N} a_n \cos(d_n u_k + \phi_n) = 0 \\ &\sum_{n=1}^{N} a_n \sin(d_n u_k + \phi_n) = 0 \ . \end{aligned}\right\} \quad k = 1, 2, \ldots, M \tag{C3}$$

Taking the gradients of f and the constraint functions, and equating components in Eq. (C2) yields

$$a_n \sin \phi_n = \frac{1}{4}\left[-\sum_{k=1}^{M} \lambda_k a_n \sin(d_n u_k + \phi_n) + \sum_{k=1}^{M} \mu_k a_n \cos(d_n u_k + \phi_n)\right] \ .$$

Summing over n and using Eqs. (C3) then gives Eq. (C1).

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